Midterm Introduction to Logic
Practice

☞ Only write your student number at the top of the exam, not your name. Also put your student number at the top of any additional pages.

☞ Put the name of your tutorial group (A, B, C, D, E, F, or G) at the top of the exam.

☞ Leave the first ten lines of the first page blank (this is where the calculation of your grade will be written).

☞ Use a blue or black pen (so no pencils, red pen or marker).

Good Luck!

1: translation into propositional logic   Translate the following sentences into *propositional logic*. Atomic sentences are represented by uppercase letters. Do not forget to provide the translation key.

  a. The doctor prescribes vitamin D only if you are either under three or over 80 years old.

  b. Despite the fact that it keeps raining, she is neither angry nor afraid.

  c. It is not the case that both the midterm and the lecture are on Wednesday or Friday.

2: translation into first-order logic)   Translate the following sentences to *first-order logic*. Do not forget to provide the translation key (one key for the whole exercise). Represent as much logical structure as possible.

  a. Anand and Bhaskar know each other only if Bhaskar loves Harry Potter.

  b. Even though Bhaskar loves neither Anand nor Deepika, he is happy.

  c. Anand likes Harry Potter more than he likes himself unless Bhaskar likes Anand more than he likes Harry Potter.

3: formal proofs   Give formal proofs of the following inferences. Don't forget to provide justifications. You can only use the Introduction and Elimination rules and the Reiteration rule.

  a. $P \to Q$
      $R \to S$

      $(P \lor R) \to (S \lor Q)$

  c. $\neg Q \lor R$

      $\neg (Q \land P \land \neg R)$

  b. $\neg P(a) \to P(b)$
      $\neg P(b)$

      $\neg (b = a)$

  d. $P \leftrightarrow \neg Q$

      $(Q \land R) \leftrightarrow (\neg P \land R)$

4: Normal forms of propositional logic

  a. Provide a negation normal form (NNF) using as connectives only $\land, \lor, \neg$ of the sentence: $(P \leftrightarrow \neg Q) \leftrightarrow (\neg R \to P)$. Show your intermediate steps.

  b. Provide a conjunctive normal form (CNF) of the sentence: $\neg (P \lor \neg Q) \lor (R \land S \land T)$. Show your intermediate steps.

5: Set theory   Given are the following three sets: $A = \{1\}$, $B = \{0, \{1, 5\}\}$ and $C = \{1, 5\}$. For each of the following statements, determine whether it is true or false. You are not required to explain the answer.

a. $1 \subseteq B$

b. $B \cap C = \emptyset$

c. $A \subseteq C \setminus B$

d. $(A \cup B) \cap C = A$

e. $B \setminus C \subseteq A$

f. $C \subseteq B$

g. $\emptyset \in A \cap B$

h. $(A \cup C) \in B$

i. $5 \in B$

j. $C \subset C \setminus (B \setminus A)$

Bonus question   Give a formal proof of the following inference. Don't forget to provide justifications. You can only use the Introduction and Elimination rules and the Reiteration rule.

$$\begin{array}{|l} \neg A \to (B \wedge C) \\ \neg A \to (\neg B \vee \neg C) \\ \hline A \end{array}$$